Self-similarity and Reynolds number invariance in Froude modelling

Romanesco broccoli

Dr. Valentin Heller (www.drvalentinheller.com)

Geohazards and Earth Processes Research Group

Group Lunch Talks, University of Nottingham

20th April 2016

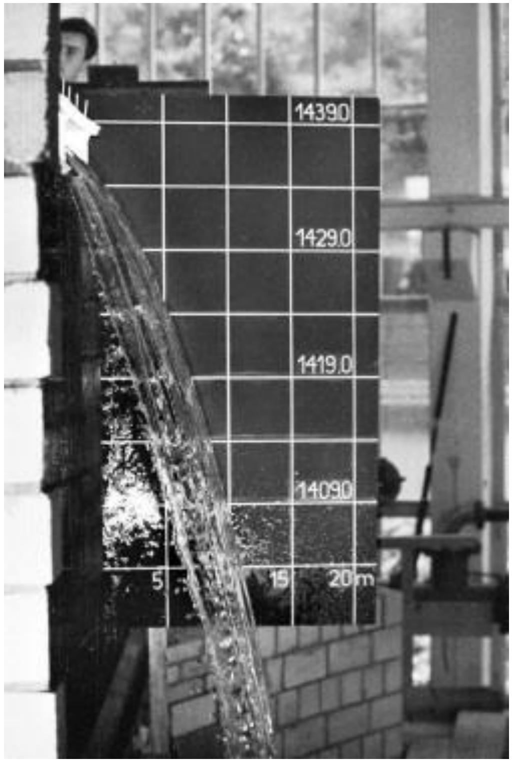
Table of content*

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

*The presentation is based on: Heller, V. (2016) Self-similarity and Reynolds number invariance in Froude modelling. *Journal of Hydraulic Research* 55(1), 1–17.

Example of scale effects

Model

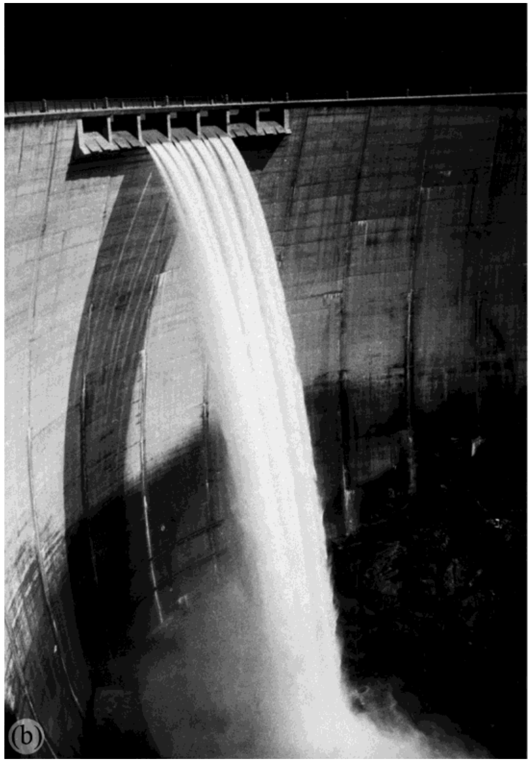


$1:\lambda = 1:30$

Jet trajectory ✓

Air concentration ✗

Real-world prototype



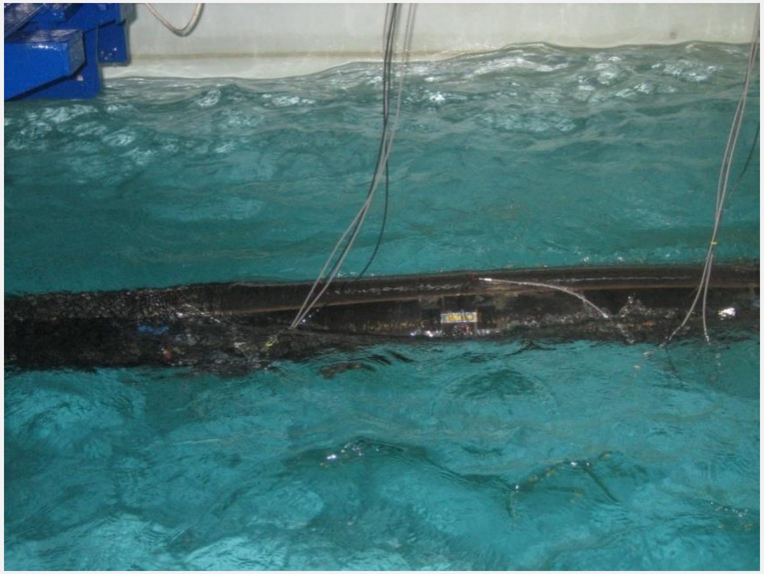
Scale ratio or scale factor $\lambda = L_P/L_M$ with L_P = characteristic length in prototype and L_M = corresponding length in model

1 Introduction

Froude similarity $F_M = F_P$

Froude number $F = V/(gL)^{1/2}$ with L = characteristic length and V = characteristic velocity

Most hydraulic phenomena are modeled after Froude, in particular free surface flows (hydraulic structures, waves, wave energy converters, etc.)



Model of Anaconda wave energy converter

Model of a hydraulic jump

Froude similarity

F is the square root of **inertial to gravity force**; i.e. in Froude models the interplay of inertial and gravity force is correctly modelled

Problem: In Froude models, the **Reynolds number** R (inertial to viscous force) and the **Weber number** W (inertial to surface tension force), etc., are **incorrectly modelled**

These R and W result in **scale effects**, which are commonly **excluded** with a **limiting R and/or W** (corresponding to a certain model size)

However, **why** can…

(i) significant scale effects be ruled out with a limiting R?

(ii) short, highly turbulent phenomena (hydraulic jumps, wave breaking), which are affected by **inertial, gravity and viscous forces**, be modelled with Froude similarity?

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

Aims

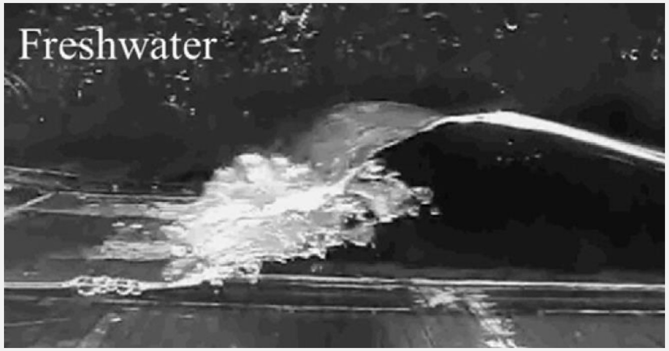
Two reviewed phenomena help to **avoid significant scale effects**:

(i) Self-similarity and

(ii) R invariance

This work aims to support Froude modelling for phenomena where **both** F and R are a priori **relevant**:

- Wave breaking
- Dike breaching
- Turbulent flows
- Hydraulic jumps
- Sediment transport
- Wakes in rivers and waves
- High-velocity open channel flows
- Plumes and jets entering rivers and wave, etc.



Wave breaking as an example where both F and R are relevant

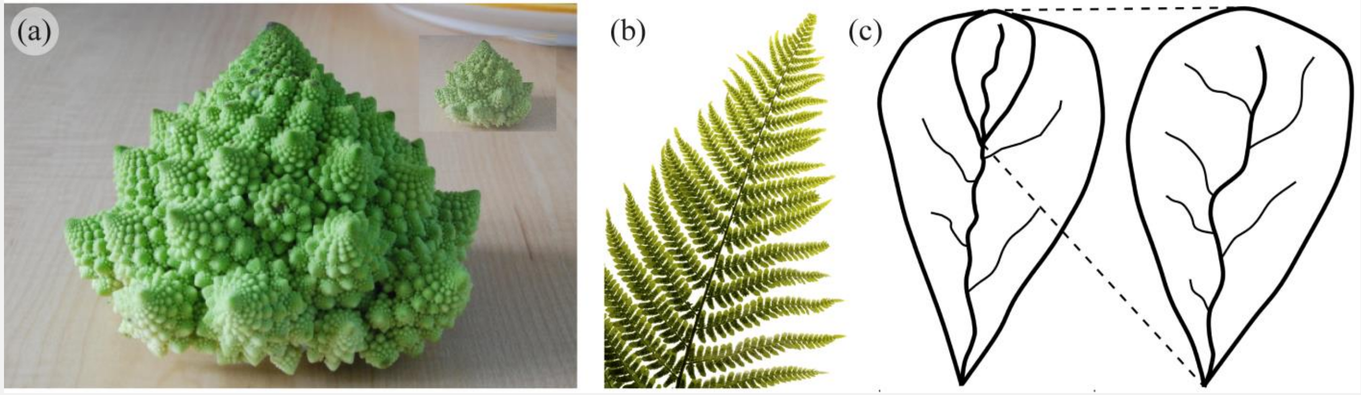
1 Introduction

(i) Self-similarity

A time-developing (or spatial) phenomenon is called self-similar if the spatial distribution of its properties at various different moments of time (or spatial locations) is obtained from one another by a similarity transformation

Self-similar profiles of velocity (or any other quantity) can be **brought into congruence** by simple scale factors which depend on only one of the variables such as location *x* or time *t*

Many features in nature and everyday life including the geometry of river networks and laws in finance **are self-similar**



Examples of geometrical self-similarity in nature: (a) Romanesco broccoli, (b) fern and (c) river networks

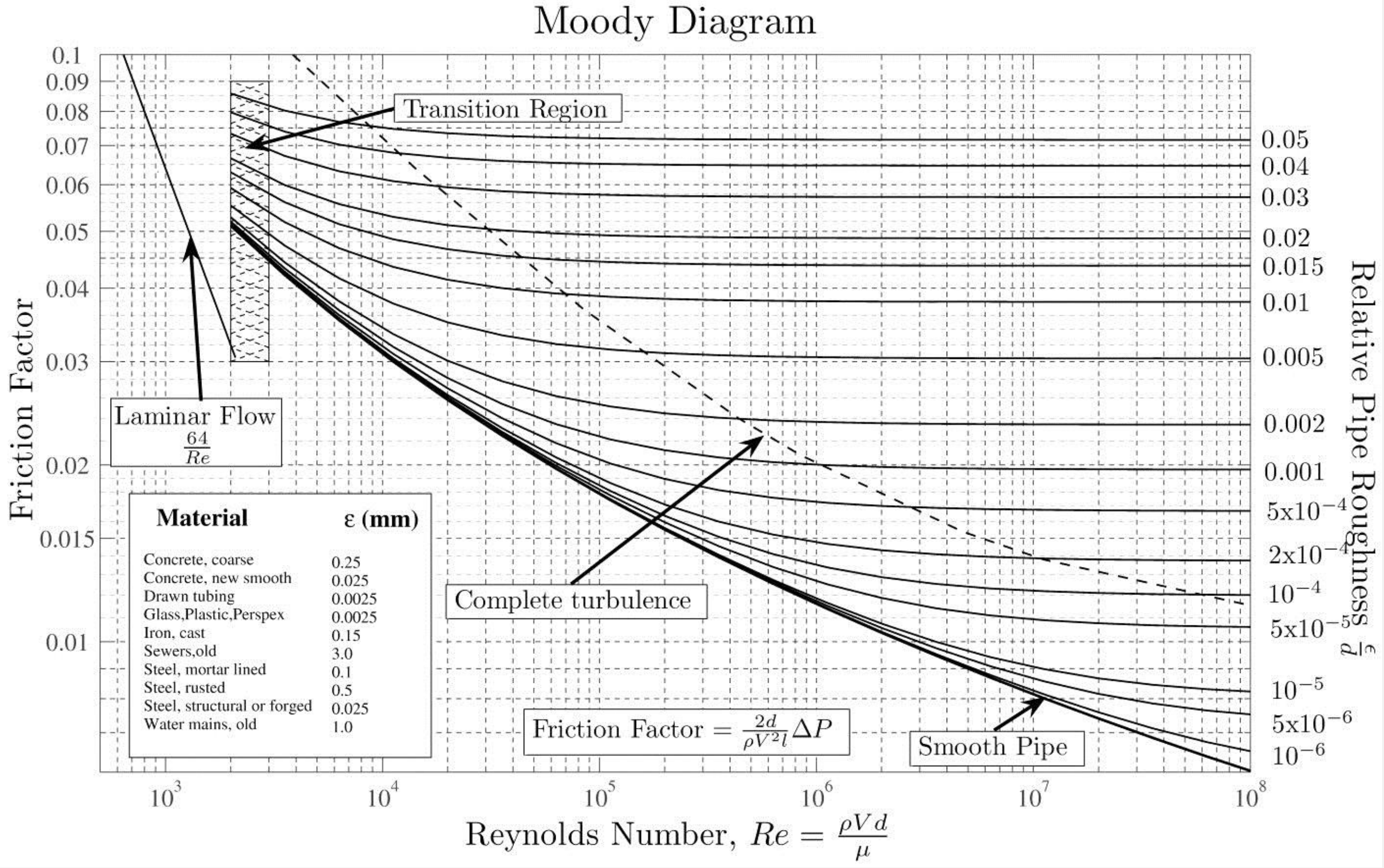
(i) Self-similarity

Self-similar conditions are based on **symmetry analysis**

The identification of self-similar flow features is **desirable because**…

- they are **universal applicable**, independent of the moment in time and/or spatial location,
- they are **simple to compute** as self-similar flows are commonly based on an **ordinary** differential equation rather than a **partial** differential equation,
- they require a **reduced** volume of **experimental work** and/or simplify data processing,
- their underlying **data points collapse to a single curve** or surface, and
- they are often **scale-invariant** such that small and cost efficient models apply.

(ii) R invariance: Example Moody diagram



R invariance in Moody diagram: The friction factor becomes R invariant for R → ∞

(ii) R invariance: Some hints why it occurs

R invariance is based on **symmetry analysis** as well and exclusively observed in high R turbulence (in contrast to self-similarity)

R invariance **directly implies scale invariance** (no source of scale effects)

R $\rightarrow \infty$ corresponds to a **vanishing effect of viscosity** ($\nu \rightarrow 0$) and/or a **large scale motion** ($L \rightarrow \infty$ and/or $V \rightarrow \infty$)

The NSEs are symmetrical (invariant) to certain operation (e.g. relative to a translation in time); for an incompressible fluid under periodic boundary conditions the **NSEs are invariant** to an operation (**spatial scaling**):

$$t, \mathbf{x}, \mathbf{v} \rightarrow \lambda^{1-m} t, \lambda \mathbf{x}, \lambda^m \mathbf{v} \qquad \text{with } \lambda \in \Re_+, m \in \Re \text{ and } \nu = 0$$

t = time, $\mathbf{x} = (x, y, z)$ = position vector and $\mathbf{v}$ = velocity, m = scaling exponent

Note: $m = 1/2$ corresponds to **Froude** and $m = -1$ to Reynolds similarity

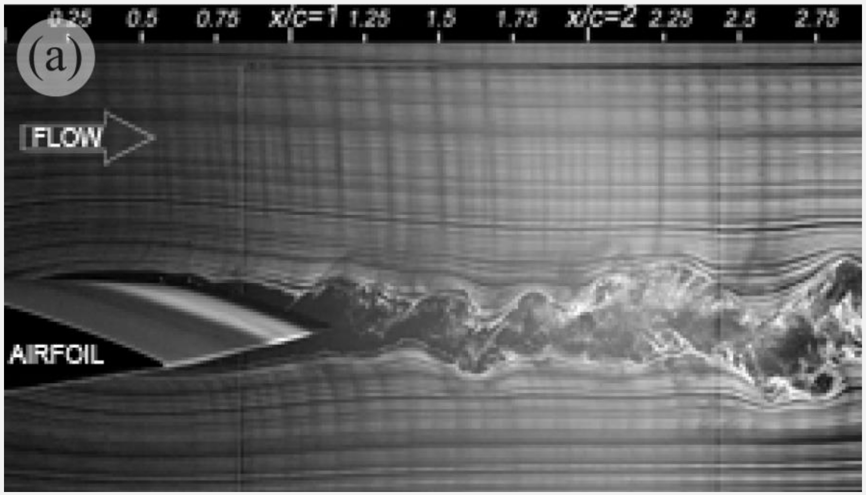
The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

Differences and similarities between (i) and (ii)

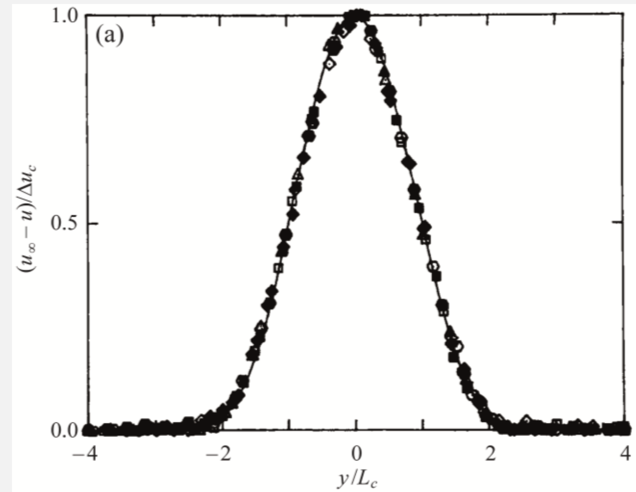
Criterion	SS	RI
Base of concept	Symmetry analysis	Symmetry analysis
Restriction to fluid flows	No (but mainly reviewed for fluid flows herein)	Yes
Restriction to high R flows	No (but often assumed and mainly reviewed for high R herein)	Yes
Correspondence to scale invariance	Not necessarily, e.g. if observed relative to a time or velocity scale rather than a length scale	Yes
Reduction of number of independent variables	Yes (length scale, time scale, velocity scale, etc.)	Yes (R)
Idealised asymptotic condition	Yes	Yes
Dependent on initial conditions	Yes	Yes
Obstructed in vicinity of solid boundaries	Yes	Yes
Useful to exclude significant scale effects	Yes	Yes

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

Self-similar phenomena: Wakes



Wake downstream of an airfoil in a wind tunnel



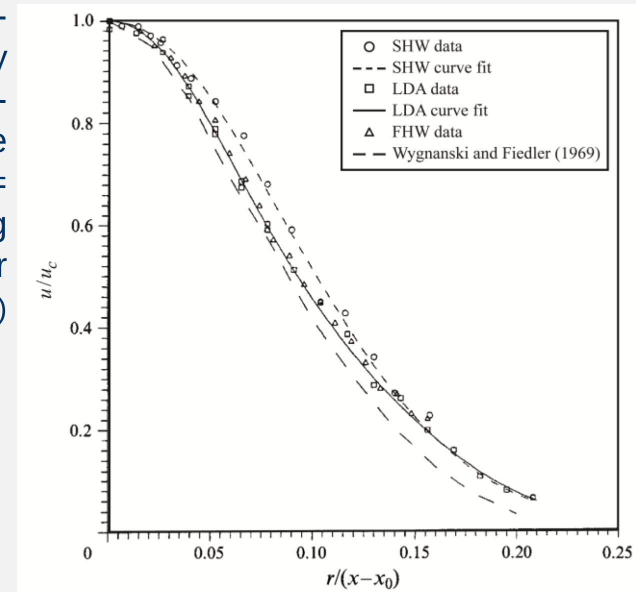
Self-similarity at solidity screen: normalised mean velocity defect versus normalised cross-flow coordinate; u = velocity, u_∞ = free stream velocity, Δu_c = velocity defect on centre line and L_c = distance centre line to cross-flow position y where $0.5\Delta u_c$ (Wygnanski et al. 1986)

- Wakes are observed **downstream of many structures** in hydro- or aerodynamics (aerofoils (photo), bridge piers, risers, etc.)
- Many of these wakes are observed in free surface flows (open channels, rivers, waves), which are **commonly modelled after Froude**
- The **data** are self-similar because they **collapse to a single curve**

Self-similar phenomena: Jets and plumes

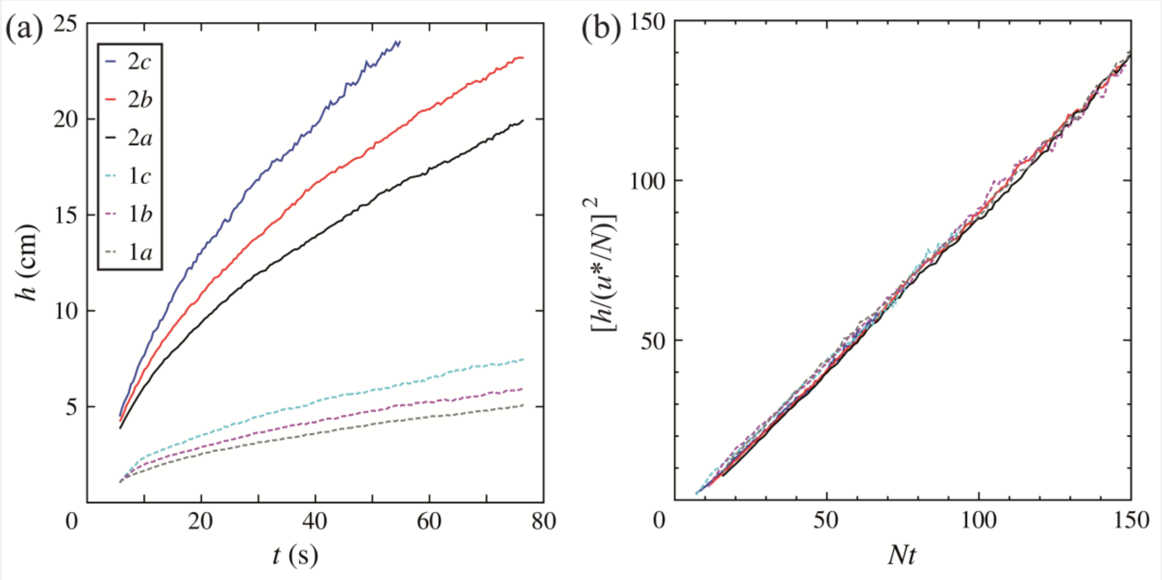
Volcanic plume

Mean velocity profile of axisymmetric jet with centreline velocity u_c; u = velocity, r = radial coordinate, x = streamwise coordinate and x_0 = virtual origin, SHW = stationary hot-wire, FHW = flying hot-wire and LDA = laser-Doppler anemometry (Hussein et al. 1994)



- **Plumes** arise from smoke, effluent from pollution outlets, seafloor hydrothermal vents and explosive volcanic eruptions (left) and are dominated by **buoyancy** at the source
- **Jets** include water jet fountains, water cannon for firefighting or jet pack dominated by **momentum** at the source
- Self-similarity results again in the **data collapse to a single curve**

Self-similar phenomena: Shear-driven entrainment



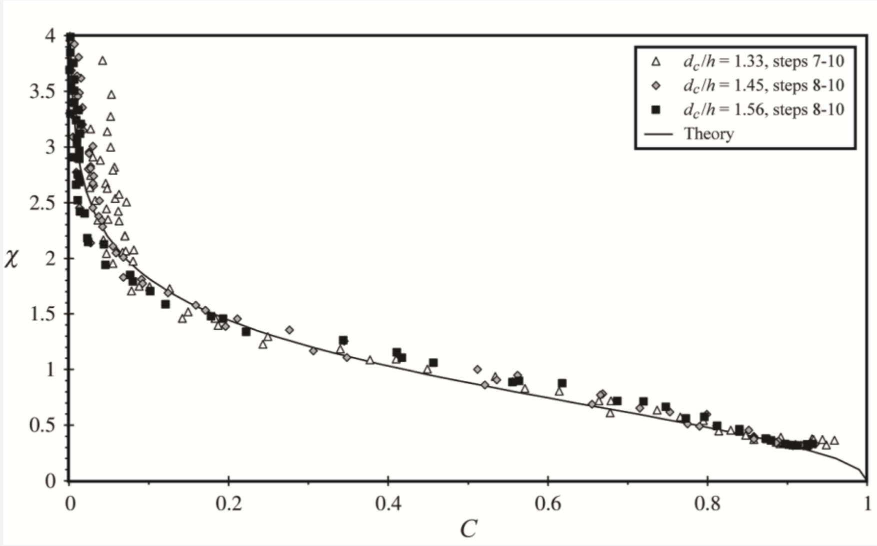
Shear-driven boundary layer growth into a linearly stratified fluid: (a) mixed layer depth evolution *h*(*t*) for six experiments and (b) collapse of data on a straight line in dimensionless form; *N* = buoyancy frequency and *u** = shear velocity (Jonker et al. 2013)

- Relevant for **deepening** of oceanic **boundary layers** due to surface winds and bottom boundary layer development on spillways
- Date above were obtained with a **direct numerical simulation**
- Self-similarity results again in a **data collapse** to a single line

2 Examples

Self-similar phenomena: High-velocity open channel flows

Turbulent air-water mixture on a chute



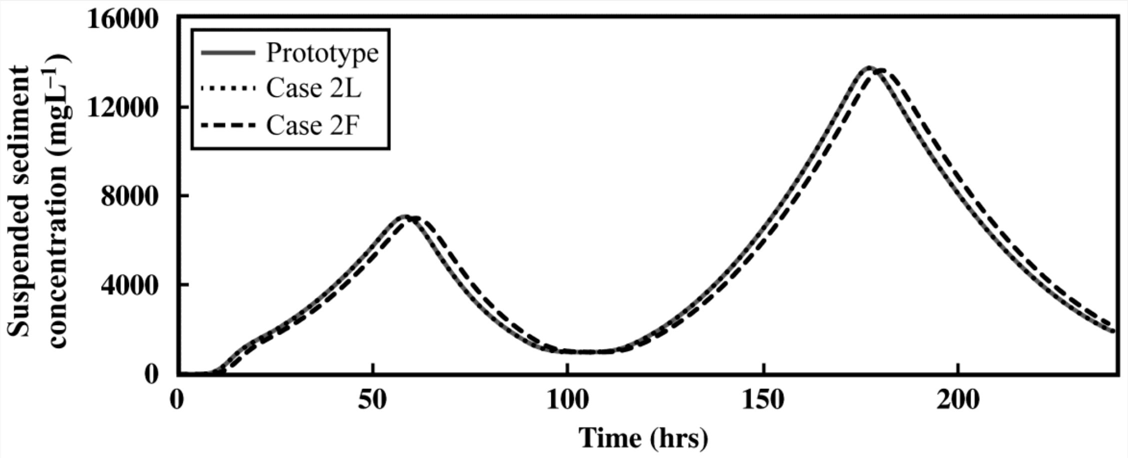
Air-water skimming flow on a stepped chute described with analytical solution (Theory): dimensionless void fraction distribution $C(\chi)$ with C = void fraction and χ = dimensionless parameter (Chanson and Carosi 2007)

- Observed on **hydraulic structures** such as spillways and chutes (left)
- Date on the right were obtained in a physical **Froude model study**
- Self-similarity results again in a **data collapse** to a single line

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

Self-similar phenomena: Sediment transport

Sediment in the Rhone River entering Lake Geneva



Suspended sediment concentration over time for prototype values (Prototype), for up-scaled test case based on Lie group scaling (Case 2L) and on traditional Froude modelling (Case 2F) (Carr et al. 2015)

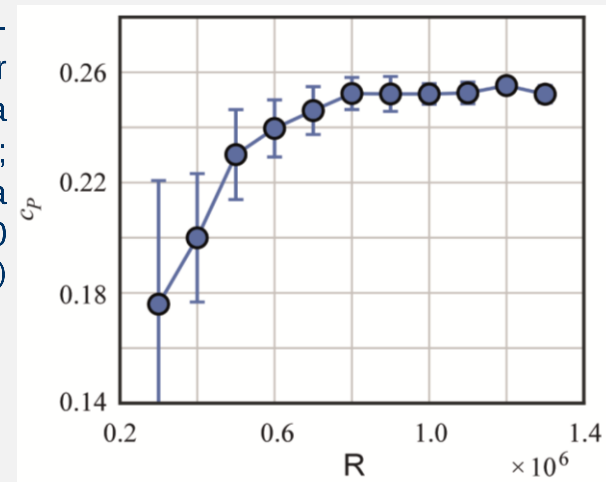
- Relevant in areas such as **fluvial hydraulics** and **coastal engineering**
- **Lie Group scaling** has been applied to the governing equations, which is an analytical transformation resulting in scaling laws different from Froude modelling laws
- Perform **better than Froude modelling** because the sediment density and grain density remain correctly scaled (contrary to Froude modelling)

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

R invariant phenomena: Tidal energy converters TECs

Horizontal axis tidal turbine

Asymptotically approached R invariant power coefficient c_P level for a tidal energy converter; figure suggests a minimum R = 800,000 (Bachant and Wosnik 2016)

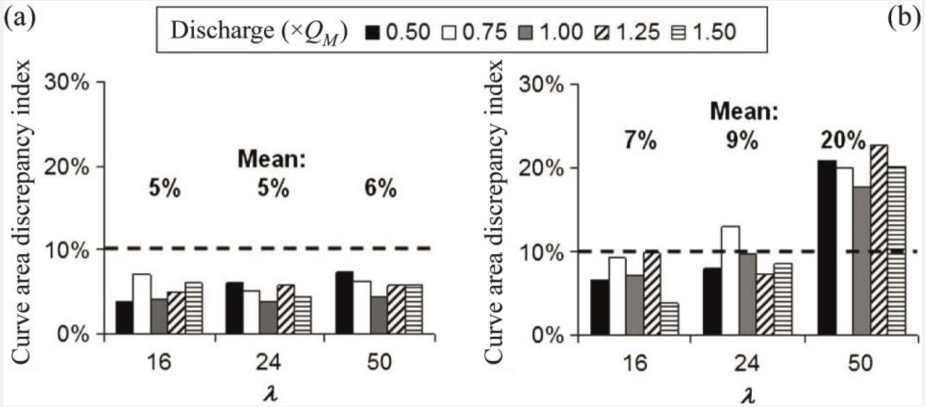


- Tens of **tidal energy converters** (left) are currently under research and development and the **UK is leading** due to excellent resources
- Physical modelling is **challenging**; R is most relevant, but results in unpractical scaling laws (e.g. velocity $v_M = \lambda \cdot v_P$)
- **Strategy:** model correct tip speed ratio and use R as large as possible; the **results** of TECs are commonly **not very reliable** (scale effects)

2 Examples

R invariant phenomena: Complete mixing in contact tanks

Solute transport in a chlorine contact tank



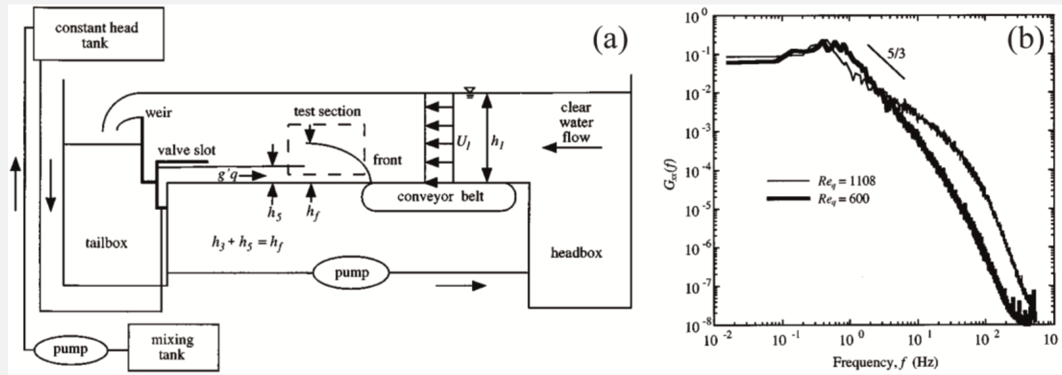
Complete mixing in a contact tank: variation of curve area discrepancy index with scale and discharge for (a) complete mixing and (b) plug flow (Teixeira and Rauen 2014)

- Commonly used to **disinfect drinking water** prior to distribution (left)
- Important are **mixing processes** and this is either achieved under complete mixing (fully turbulent) or plug flow (not fully turbulent)
- Physical model study was conducted at different scales (scale series) and results are compared; **complete mixing** resulting in **insignificant** and **plug flow** in **significant scale effects** for $\lambda > 24$ (right)

2 Examples

R invariant phenomena: Gravity currents

Gravity current in the atmosphere in Khartoum, Sudan



Gravity current investigated with (a) set-up based on arrested gravity current method and (b) power spectra $G_{xx}(f)$ revealing deviations of low from high R flow data measured in most energetic region at current front (Parsons and García 1998)

- These are **buoyancy driven** fluid flows moving due to density differen-ces (temperature, suspended material) primarily in the horizontal direction
- Relevant for thunderstorm outflows, sea-breeze fronts, river front mixing with sea water in estuaries, snow avalanches, turbidity currents, etc.
- Tests were conducted **at one point** in gravity current front showing –5/3 law (which strongly suggests self-similarity)

2 Examples

Phenomena and quantities involving **self-similarity** at large R with limitations and references

Phenomenon	Self-similar invariant quantity	Investigated R range	Comment	Reference
Axisymmetric jet	Front position and spread of radial integral of ensemble-averaged concentration of passive scalar transport	$2M_0^{1/2}/\nu = 4{,}815$	Ensemble average over 16 DNS tests; negligibility of viscous effects was confirmed with additional tests at $2M_0^{1/2}/\nu = 6{,}810$	Craske et al. (2015)

Phenomenon	Self-similar invariant quantity	Investigated R range	Comment	Reference
Axisymmetric jet	Front position and spread of radial integral of ensemble-averaged concentration of passive scalar transport	$2M_0^{1/2}/\nu = 4{,}815$	Ensemble average over 16 DNS tests; negligibility of viscous effects was confirmed with additional tests at $2M_0^{1/2}/\nu = 6{,}810$	Craske et al. (2015)
Axisymmetric jet	Mean velocity profile (Fig. 5), centreline velocity and many higher order moments and velocities	$du/\nu = 100{,}000$	$z/d > 50$ (not everybody agrees, see e.g. Carazzo et al., 2006)	Hussein et al. (1994)
Axisymmetric wake downstream of a sphere	Mean velocity defect and turbulent velocity fluctuation profiles	$du_\infty/\nu = 8{,}600$	$50 < x/d < 150$, data collapse to a curve if normalised with the characteristic velocity scale $u_\infty d^{2/3}(x - x_0)^{-2/3}$ and the characteristic length scale $(x - x_0)^{1/3}d^{2/3}$	Uberoi and Freymuth (1970)
Axisymmetric wake downstream of a spherical body	Mean velocity defect profile	$\Delta u_c(x = 0)[\Delta Q/(2\pi\Delta u_c(x = 0))]^{1/2}/\nu = 2{,}171$	Universal SS for $x > 5000d$	Redford et al. (2012)
Pipe flow	Velocity distribution over cross-section	Fully turbulent	Universal SS applies to smooth and rough circular straight pipes	Taylor (1954)
Planar jet	Mean velocity and velocity fluctuation profiles	$\Delta yu/\nu = 30{,}000$	$x/d > 40$	Gutmark and Wygnanski (1976)
Planar mixing layer	Mean velocity and velocity fluctuation profiles	Not available, nozzle exit speed 8 m/s	Self-similar profiles are not universal, as profiles may depend on the state of the initial boundary layer and/or the initial flow conditions	Champagne et al. (1976)

Phenomenon	Self-similar invariant quantity	Investigated R range	Comment	Reference
Planar wake downstream of a circular cylinder, screens and solid strip, flat plate and symmetrical aerofoil	Mean velocity defect profiles (Fig. 4)	$1{,}360 \leq \Delta yu_\infty/\nu \leq 6{,}500$	Velocity defects 0.03 - $0.15u_\infty$; SS within individual wake generators, but no universality; measurements at $100 \leq \theta \leq 2{,}000$	Wygnanski et al. (1986)
Plumes	Mean velocity profile	$du/\nu \geq 600$	For $z/d > 50$ the parameter α_e becomes constant	Carazzo et al. (2006)
Rotor wake vortex	Vorticity and azimuthal velocity profiles	$500 \leq du/(2\nu) \leq 2{,}000$	Based on DNS	Ali and Abid (2014)
Shear-driven entrainment into a linearly stratified fluid	Growth of boundary layer (Fig. 6), mean buoyancy, mean horizontal velocity, buoyancy flux, momentum flux	$36 \leq u^{*2}/(N\nu) \leq 1{,}214$	Proof of entrainment law $e/u^* \propto \mathrm{Ri}^{-1/2}$ with dimensional arguments	Jonker et al. (2013)
Turbulence in quasi-2D	PDFs for longitudinal velocity differences	Sufficient high	For inertial subrange $E(k) \sim \kappa^{-5/3}$ (Kolmogorov –5/3 spectrum)	Kolmogorov (1941), Pope (2000)
Turbulence in quasi-2D rapidly rotating fluid	PDFs for longitudinal velocity differences	$\lambda_g\overline{u'}/\nu = 360$	For inertial subrange $E(k) \sim \kappa^{-2}$ (anomaly to Kolmogorov –5/3 spectrum)	Baroud, Plapp, She, and Swinney (2002)
Turbulent open channel air-water flow on a stepped chute	Distributions of void fraction (Fig. 7), bubble count rate, interfacial velocity and turbulence level	$380{,}000 \leq d_h u/\nu \leq 710{,}000$		Chanson and Carosi (2007)

2 Examples

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

Phenomena and quantities involving **R invariance** with limitations and references

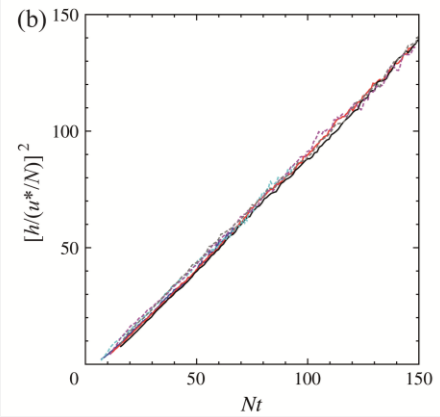
Phenomenon	R invariant quantity	Investigated R range	Comment	Reference
Cross-flow turbine	Mean power coefficient (Fig. 11)	$du_\infty/\nu \geq$ 800,000	Tests conducted for 300,000 $\leq du_\infty/\nu \leq$ 1,300,000	Bachant and Wosnik (2016)
Gravity current	Mixing processes in current front (Fig. 13)	1,000 $\leq h_5(g'q)^{1/3}/\nu \leq$ 2,012	Applies to most energetic region; no RI for $h_5(g'q)^{1/3}/\nu <$ 1,000	Parsons and García (1998)
Pipe flow	Energy head losses (Moody diagram)	$du/\nu \geq$ 50,000	This R value applies for $k/d \geq 0.07$, but R needs to be larger for $k/d < 0.07$	Massey (1989)
Rough horizontal cylinder in steady flow	Drag coefficient $C_D \approx 1.2$	75,000 $\leq du/\nu \leq$ 480,000	d = 0.21 and 0.5 m, effective roughness k/d = 0.038	Chaplin and Subbiah (1997)
Shields diagram	Critical Shields stress	$d_g u^*/\nu > 400$		Shields (1936)
Wall-bounded turbulence	Mean flow and Reynolds shear stresses in 2D channel flow	1,000 $\leq wu^*/(2\nu) \leq$ 6,000		Schultz and Flack (2013)
Wall turbulence	Wake factor in the law of the wall/wake	$\theta u_\infty/\nu \geq$ 15,000; $du/\nu \geq$ 400,000	The first criterion is for boundary layers and the second for pipe flow	Smits, McKeon, and Marusic (2011)
Water treatment tank (contact tank under complete mixing)	Dispersion and mixing processes (Fig. 12)	Not available	Turbulent flow for R > 4,000	Teixeira and Rauen (2014)

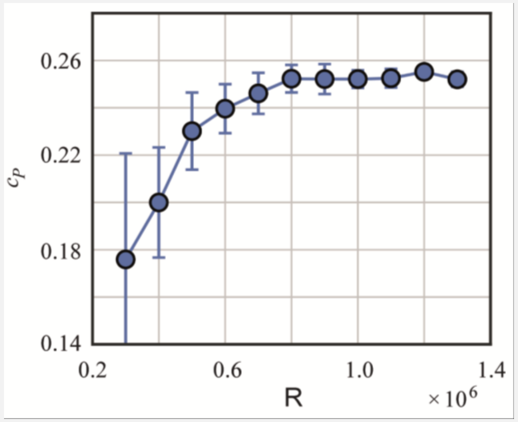
3 Over-shadowing

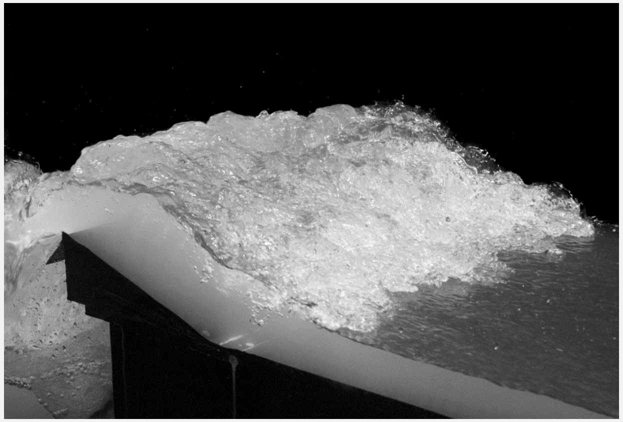
Self-similarity does not guarantee that such a motion is actually dominant in a flow; it may be **over-shadowed** by other, more dominant effects (e.g. shear-driven entrainment was investigated under idealised conditions)

Self-similarity is an **idealised asymptotic condition** after the initial conditions are over-come requiring potentially a long time or distance, such that self-similarity may never be reached (e.g. in plumes and jets)

Other force ratios may also introduce scale effects, and they may interfere with features a priori believed to be R invariant (e.g. W resulting in larger air bubbles in hydraulic jumps which may indirectly affect energy dissipation)







3 Over-shadowing

The **conditions** under which self-similarity and R invariance were observed need to be considered carefully; it may only apply to a **particular region** of the flow, or a **particular parameter** (see previous tables)

Phenomena involving **biological or chemical processes** (e.g. water and wastewater treatment tanks) require a **certain amount of time** for the reactions or processes to take place, irrespective of whether the turbulent mixing processes are self-similar

Despite of these limitations, self-similarity and R invariance are **important concepts** to understand why significant scale effects may be excluded in Froude models with a limiting R

These concepts are hoped to **support the design and execution of many future Froude studies**

4 Conclusions

- This work aims to **supporting Froude modelling** for phenomena where both the Froude number and the Reynolds number R are a priori relevant
- The two concepts **(i) self-similarity** (at large R only) and **(ii) R invariance** have been illustrated
- These concepts **explain** (a) why significant scale effects in Froude models can be ruled out with a limiting R and (b) why short, highly turbulent phenomena can be modelled after Froude
- A **wide range of fluid phenomena** involving self-similarity at large R and R invariance **were reviewed**
- **Tables** summarise many phenomena involving (i) and (ii), and are **hopped to support many future Froude studies**

Thank you for your attention!

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

Acknowledgement

Dr Maarten van Reeuwijk is acknowledge for helpful discussions and critical comments. This work was initiated during an Imperial College London Research Fellowship.

References

- **Heller, V. (2016). Self-similarity and Reynolds number invariance in Froude modelling, *J. Hydraulic Res.* 55(1), 1–17.**
- Bachant, P., Wosnik, M. (2016). Effects of Reynolds number on the energy conversion and near-wake dynamics of a high solidity vertical-axis cross-flow turbine. *Energies* 9(2), 1-18.
- Carr, K.J., Ercan, A., Kavvas, M.L. (2015). Scaling and self-similarity of one-dimensional unsteady suspended sediment transport with emphasis on unscaled sediment material properties. *J. Hydraulic Eng.-ASCE* 141(5), 04015003-1–9.
- Chanson, H., Carosi, G. (2007). Turbulent time and length scale measurements in high-velocity open channel flows. *Exp. Fluids* 42(3), 385–401.
- Craske, J., Debugne, A.L.R., van Reeuwijk, M. (2015). Shear-flow dispersion in turbulent jets. *J. Fluid Mech.* 781, 28–51.
- Hussein, H.J., Capp, S.P., George, W.K. (1994). Velocity measurements in a high-Reynolds-number, momentum-conserving, axisymmetric, turbulent jet. *J. Fluid Mech*. 258, 31–75.
- Jonker, H.J.J., van Reeuwijk, M., Sullivan, P.P., Patton, E.G. (2013). On the scaling of shear-driven entrainment: a DNS study. *J. Fluid Mech.* 732, 150–165.
- Parsons, J.D., García, M.H. (1998). Similarity of gravity current fronts. *Phys. Fluids* 10(12), 3209–3213.
- Teixeira, E.C., Rauen, W.B. (2014). Effects of scale and discharge variation on similitude and solute transport in water treatment tanks. *J. Environ. Eng.-ASCE* 140(1), 30–39.
- Wygnanski, I., Champagne, F., Marasli, B. (1986). On the large-scale structures in two-dimensional, small-deficit, turbulent wakes. *J. Fluid. Mech.* 168, 31–71.

Dr. Valentin Heller (www.drvalentinheller.com)